Chapter 02

Basics of Financial Mathematics and Investment:

Financial mathematics and investment are essential to understanding how money grows or shrinks over time. These concepts help in making informed decisions about savings and investments. The key concepts include simple interest, compound interest, present value, and future value. Let's go over each one in detail.

1. Simple Interest

Simple interest is a method of calculating interest only on the original principal amount, without considering interest accrued from previous periods.

$$I = P.r.t$$

- I: Simple interest.
- P: Principal amount (initial investment).
- r: Annual interest rate (as a decimal).
- t: Time or duration (in years).

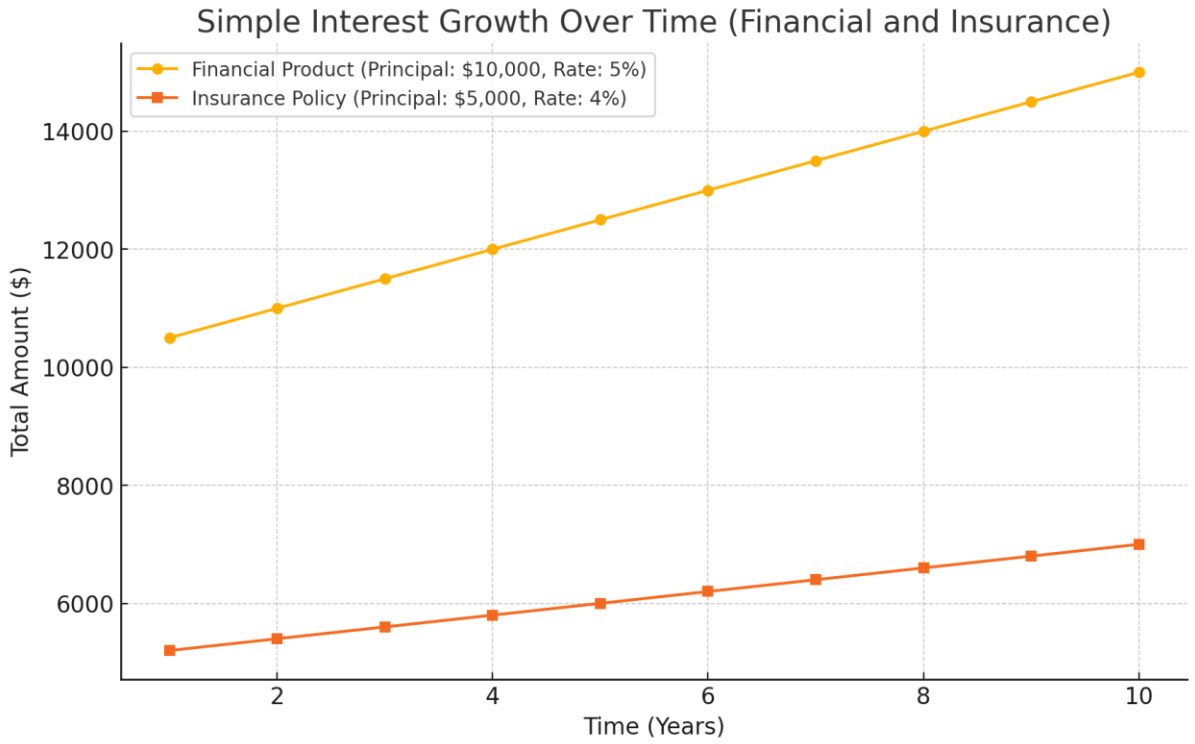
Example:

If you deposit $1000 in a simple interest account with a 5% interest rate for 3 years:

$$I = 1000 \times 0.05 \times 3 = 150 \text{ dollars.}$$

Total amount after 3 years = Principal + Interest:

$$1000 + 150 = 1150 \text{ dollars.}$$



Here is the simulation graph showing the growth of Simple Interest over time for both a financial product and an insurance policy:

- **Blue Line (Financial Product):** Principal of $10,000 with a 5% interest rate. You can see how the total amount grows steadily over 10 years.
- **Orange Line (Insurance Policy):** Principal of $5,000 with a 4% interest rate. It also grows over the same period, but at a slower rate due to the lower principal and interest rate.

The lines represent the accumulation of the principal plus interest over time. As you can observe, the higher the principal and interest rate, the steeper the growth.

2. Compound Interest

Compound interest takes into account the interest earned in previous periods, where the interest is added to the principal, and the next interest is calculated on the total (principal + interest).

$$A = P\left(1 + \frac{r}{n}\right)^{\{n \times t\}}$$

- A : Final amount after compounding.
- P : Principal amount (initial investment).
- r : Annual interest rate.
- n : Number of times interest is compounded per year (e.g., 1 for annual, 12 for monthly).
- t : Time or duration (in years).

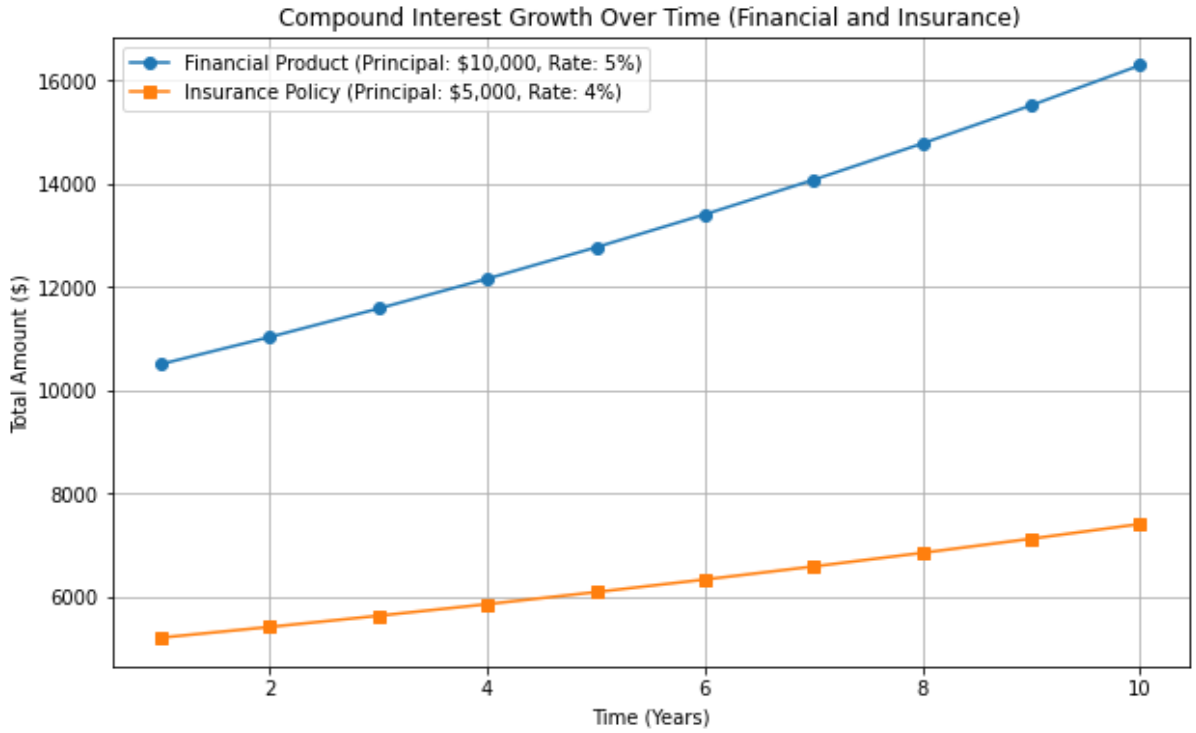
Example:

Let’s simulate the compound interest growth for both financial and insurance scenarios.

1. **Financial Product:**
    - **Principal (P):** $10,000
    - **Rate of Interest (R):** 5% per annum
    - **Time (T):** 1 to 10 years
2. **Insurance Policy:**
    - **Principal (P):** $5,000
    - **Rate of Interest (R):** 4% per annum
    - **Time (T):** 1 to 10 years

I'll now generate graphs based on this compound interest simulation.

It seems like I can’t do more advanced data analysis right now. Please try again later. If you'd like, I can explain the compound interest concept or guide you through manually calculating it



3. Future Value

Future value is the amount of money you will have in the future based on a given interest rate. It can be calculated using either simple or compound interest.

Formula (for compound interest):

$$FV = P\left(1+\frac{r}{n}\right)^{\{n\times t\}}$$

- FV : Future value.
- P : Principal amount (initial investment).
- r : Annual interest rate.
- n : Number of times interest is compounded per year.
- t : Time or duration (in years).

Example:

If you deposit $2000 with a 6% compound interest rate for 5 years:

$$FV \approx 2676.45 \, dollars$$

4. Present Value

Present value is the value of a future amount of money in today's terms, discounted at a specific interest rate. In other words, how much do you need to invest now to reach a certain amount in the future?

- PV: Present value.
- FV: Future value.
- r: Annual interest rate.
- n: Number of times interest is compounded per year.
- t: Time or duration (in years).

Example:

If you want to have $5000 in 10 years with a 4% annual interest rate, the present value you need to invest now is:

$$PV \approx 3377.05 \, dollars$$

To reach $5000 in 10 years, you need to invest approximately $3377.05 now.

Example Simulation:

Let’s consider two scenarios where the present value is calculated for a financial product and an insurance policy, assuming a future value of $10,000.

1. **Financial Product:**
    - **Future Value (FV):** $10,000
    - **Discount Rate (r):** 5% per annum
    - **Time (t):** 1 to 10 years

2. **Insurance Policy:**
    - **Future Value (FV):** $10,000
    - **Discount Rate (r):** 4% per annum
    - **Time (t):** 1 to 10 years

I'll now generate the simulation graphs for present value over time for both cases.