Chapter 03

Actuarial science, especially in the field of Insurance, is a specialized branch of mathematics that uses statistical and probabilistic methods to assess risk and determine premiums and reserves for insurance policies. Here's an in-depth look at the foundational aspects of actuarial science in insurance, including key concepts and methods for calculating net and gross premiums for life insurance contracts.

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1. Actuarial Science in the Field of Insurance

Actuarial science applies mathematical, statistical, and financial theories to solve problems in insurance and finance, especially around risk assessment and management. Actuaries work to analyze uncertain future events, particularly in the context of:

- **Life insurance**: Ensuring financial protection against death, longevity risks, and associated financial burdens.
- **Health insurance**: Modeling claims and healthcare risks to set premium levels and manage health-related risks.
- **General (non-life) insurance**: Calculating risk for property, casualty, and liability insurance products.
- **Pensions and annuities**: Determining funding levels and benefits for retirement and annuity products.

In the insurance industry, actuarial science supports the development and pricing of products, helps with financial reporting and solvency assessments, and plays a significant role in regulatory compliance.

Key Concepts in Actuarial Science for Insurance:

1. **Mortality Tables**: These tables estimate the probability of death for individuals of different ages, and they form the basis for life insurance calculations.
2. **Life Expectancy**: Calculations using mortality tables to determine the expected remaining years of life, often central to life insurance products.
3. **Probability Theory**: Key to calculating the likelihood of claims, using discrete and continuous random variables to model different life events.
4. **Present Value and Future Value**: Essential for valuing future cash flows, considering the time value of money (e.g., discounting future benefits to today's value).
5. **Risk and Uncertainty**: Evaluating both diversifiable and non-diversifiable risks and understanding how they affect policyholder behavior and company profitability.

Mortality tables are foundational tools in actuarial science, especially for life insurance. They contain statistical data that represent the probabilities of death and survival at different ages, which is crucial for calculating premiums, reserves, and benefits in life insurance policies. Below is a detailed course outline that covers the essential aspects of mortality tables, their development, and how they are used in insurance calculations.

1.Mortality Tables in Actuarial Science

1.1 Introduction to Mortality Tables

- **Definition:** Mortality tables, also known as life tables, are statistical tables that show the probability of death or survival for a population over different ages.

- **Purpose:** They provide actuarial models for calculating the likelihood of death within a specific timeframe and are critical for life insurance, pensions, and annuity products.

- **Types of Mortality Tables:**

  - **Static (or Period) Mortality Table:** Reflects mortality rates observed in a specific period (e.g., A2000 table).

  - **Dynamic (or Cohort) Mortality Table**: Reflects changing mortality rates over time for a particular birth cohort, accounting for improvements in life expectancy.

2.1 Structure and Components of Mortality Tables

- $\mathbf{q_x}$ **(Probability of Death):** Represents the probability that a person aged **x** will die before reaching age $\mathbf{x + 1}$.

- $\mathbf{p_x}$ **(Probability of Survival):** Represents the probability that a person aged **x** will survive to age $\mathbf{x + 1}$ (i.e., $\mathbf{p_x = 1 - q_x}$).

- $\mathbf{l_x}$ **(Number of Survivors):** The hypothetical number of people alive at the beginning of each age **x**, starting from an initial cohort (e.g., 100,000 at age 0).

- $\mathbf{d_x}$ **(Number of Deaths):** The number of deaths between ages **x** and $\mathbf{x + 1}$, calculated as $\mathbf{d_x = l_x \cdot q_x}$.

- $\mathbf{T_x}$ **(Total Future Years Lived):** The total number of years lived by the cohort from age **x** onward.

- $\mathbf{e_x}$ **(Life Expectancy)**: The average remaining lifetime for individuals aged **x**, calculated as $\mathbf{e_x = T_x / l_x}$.

3. 1 Types of Mortality Tables and Their Uses

- **Life Insurance Mortality Tables:** Used for calculating life insurance premiums and reserves; they may be adjusted to reflect insured populations, which often experience lower mortality than the general population.

- **Population Mortality Tables:** Reflect general population mortality, such as the tables produced by government agencies for public health statistics.

- **Annuity Mortality Tables**: Used in pension and annuity calculations, often adjusted for higher life expectancy since annuitants generally live longer.

- **Select and Ultimate Mortality Tables:** Incorporate an initial period (select period) where mortality rates are lower due to recent underwriting and an ultimate period where mortality stabilizes.

4.1 Constructing a Mortality Table

- **Data Collection:** Mortality tables are developed using large datasets from insurance claims, health records, and census data. Data can be sourced from governmental and private sector records.

- **Mortality Rate Calculation:**

  - Calculated by dividing the number of deaths at a certain age by the number of people exposed to risk at that age.

  - Rates may be adjusted for age, sex, and other demographic factors.

- **Smoothing Techniques**: Mortality data can be irregular, especially at older ages. Actuaries often use mathematical smoothing techniques to create a more stable table, such as moving averages or graduation techniques.

- **Projection Techniques:** Dynamic tables often project future mortality improvements, using methods like the Lee-Carter or Cairns-Blake-Dowd models to account for longevity trends.

5.1. Applications of Mortality Tables in Life Insurance Calculations

Mortality tables are integral in actuarial calculations for insurance products:

5.1.1 Premium Calculation

- **Net Premium:** Uses mortality probabilities to calculate the pure cost of providing life insurance coverage.

- **Gross Premium**: The net premium plus loadings for expenses, profit, and contingencies. This is the actual amount charged to the policyholder and covers all aspects of maintaining the policy.

Steps for Calculating Net Premiums

1. **Identify the Type of Policy**: For example, term life insurance (providing coverage for a specified period) or whole life insurance (coverage for the policyholder's lifetime).
2. **Select Mortality Rates**: Use a relevant mortality table, such as the A2000 table, to identify mortality probabilities at specific ages.
3. **Determine the Interest Rate**: Choose an appropriate discount rate to calculate the present value of future benefits. This rate reflects the investment return on premium funds held by the insurer.
4. **Calculate the Expected Present Value (EPV) of Benefits**:
    - For a term policy, the EPV of benefits is calculated for each policy year based on the probability of death within that year.
    - For whole life insurance, the EPV is the present value of benefits paid at any point over the insured's life expectancy.

The **Net Premium** formula for a whole life policy can be simplified to:

$$\text{Net Premium} = \frac{\text{EPV of Benefits}}{\text{EPV of Premium Payments}}$$

For a term life policy:

$$\textbf{Net Premium} = \sum_{t=0}^{n} B \cdot v^t \cdot q_x(t)$$

where:

- **B** is the benefit amount,
- **v** is the discount factor **(v = 1 / (1 + i)),**
- $q_x(t)$ is the probability of death within year **t** for someone aged **x**.

- **Gross Premium**: Adds expense loadings and profit margins to the net premium.

Steps for Calculating Gross Premiums

1. **Add Loadings for Expenses**: Actuaries add costs associated with underwriting, administration, and claims processing.
2. **Include Profit Margins**: A certain margin may be added to cover profit and unforeseen contingencies.
3. **Calculate Gross Premium**:

$$\textbf{Gross Premium} = \textbf{Net Premium} + \textbf{Expense Loading} + \textbf{Profit Loading}$$

Gross premium calculation also accounts for **inflation** and **persistency rates** (probability of the policyholder continuing the policy without lapsing).

Example Calculations for a Simplified Life Insurance Product:

Let's say you have a 1-year term policy for a person aged 40, with a benefit of $100,000. Assuming:

- Probability of death in the year = 0.001.
- Interest rate = 5%.

1. **Net Premium**:
    - **Expected Death Benefit (EPV)**: 100,000×0.001=100.
    - **Discounted EPV**: 100/(1+0.05)=95.24100 / (1 + 0.05) = 95.24100/(1+0.05)=95.24.

So, the **Net Premium** = $95.24.

2. **Gross Premium** (with an expense loading of $10 and profit margin of $5):
   Gross Premium=95.24+10+5=110.24=110.2

5.2 Reserve Calculation

- **Purpose**: Reserves ensure that insurers have sufficient funds to cover future policy benefits.

- **Reserves Calculation Using Mortality Data:**

- Prospective Reserve: The present value of future benefits minus the present value of future premiums.

- **Retrospective Reserve**: The accumulated premiums paid minus the benefits paid up to a given point, adjusted with interest.

5.3 Benefit Calculation for Life and Annuity Products

- Mortality tables enable actuaries to calculate the expected payout for policies where benefits are contingent on the survival or death of the insured.

- For life annuities, mortality tables help in determining payouts by estimating the duration of benefit payments based on survival probabilities.

6. Calculating Expected Values Using Mortality Tables

Actuarial calculations for life insurance depend on the expected present value (EPV) of benefits, which integrates mortality probabilities over time.

6.1. Expected Value of Death Benefits:

$$\textbf{EPV of Death Benefits} = \sum_{t=0}^{n} B \cdot \left(\prod_{k=0}^{t-1} \mathbf{p}_x(\mathbf{k})\right) \cdot \mathbf{q_x(t)} \cdot \mathbf{v^t}$$

where:

- **B** is the benefit,
- $\mathbf{p}_x(\mathbf{t})$ is the probability of survival for each year up to **t**,
- $\mathbf{q_x(t)}$ is the probability of death in year **t**.

2. Expected Value of Survival Benefits:

- This calculation is crucial for annuity products where benefits are based on the insured's continued survival.
- Formula: Similar to the death benefit but focuses on survival probabilities alone.

7.1 Practical Exercises and Case Studies

- **Exercise 1**: Using a sample mortality table, calculate the net premium for a 10-year term life insurance policy for an individual aged 30.
- **Exercise 2**: Calculate the gross premium for the same policy, adding assumed loadings for expenses and profit.
- **Case Study**: Review how mortality projections affect the pricing of whole life policies versus term policies.