TD revision

REVISION

2024 /2025

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1. Quiz: Exercise1

Consider the following linear transformation:

$$T : \mathbb{R}^3 \to \mathbb{R}^3, \quad T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x \\ x+y \\ x+z \end{pmatrix}$$

Question

- Find the matrix representation of the linear transformation T.
- Find the eigenvalues of T.
- Find the eigenvectors corresponding to each eigenvalue.
- Compute the image of the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ under T.

2. Quiz: Exercise 2

The variances and covariances are given as follows:

$V(X) = 4, \quad V(Y) = 2, \quad V(Z) = 4, \quad \mathrm{COV}(X, Y) = 1, \quad \mathrm{COV}(X, Z) = 0, \quad \mathrm{COV}(Y, Z) = 1$

Question 1

1. From the given informations, form the covariance matrix.

Question 2

Find the eigenvalues, then organize them in a table with percentages and cumulative percentages.

Question 3

3. What is the percentage of variance explained by each principal component?

Question 4

4. Based on the Kaiser criterion, how many principal components should be retained?

Question 5

5. Find the eigenvector corresponding to λ_2. then deduce the normalized eigenvector corresponding to λ_2;

Question 6

Calculate the individuals components.

3. Quiz: Exercise3

Through applying the Principal Component Analysis (PCA) method to a data table X_{43} consisting of 3 ***homogeneous*** variables observed on 4 individuals, we obtained the following results, part of which is illustrated as follows:

$$g' = \begin{pmatrix} 3 & 4 & 6 \end{pmatrix}$$

$$U_1 = \begin{pmatrix} 0.46 \\ 0.88 \\ -0.08 \end{pmatrix}, \quad U_2 = \begin{pmatrix} -0.37 \\ 0.27 \\ 0.89 \end{pmatrix}$$

$$F_1 = \begin{pmatrix} -1.42 \\ 0 \\ -0.80 \\ 2.22 \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0.99 \\ 0 \\ -1.16 \\ 0.17 \end{pmatrix}$$

$$\lambda_2 = 0.61, \quad \lambda_3 = 0.05$$

Where g is the center of the individuals' cloud (gravity center), and F_2 are the coordinates of the individuals on the space spanned by the eigenvectors U_2 , corresponding to the largest two eigenvalues λ_1 and λ_2 in descending order.

Question

1. Complete the following blanks:

$$M = \begin{pmatrix} -1 & \cdots & \cdots \\ 0 & 0 & 0 \\ \cdots & -1 & \cdots \\ \cdots & \cdots & 0 \end{pmatrix}, \quad X_{43} = \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}, \quad V = \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix},$$

$$I = \cdots, \quad \lambda_1 = \cdots$$

Hint:

2. Explain how to complete the blanks above:

For the centered matrix M;

For the basic data matrix X_{43} ;

The variance-covariance matrix V;

For the total variance I;

For the first eigenvalue λ_1.