Badji Mokhtar Annaba University
Faculty of Economic Sciences, Commercial Sciences, and Management Sciences
Department of Finance

Second-Year Master's Degree in Finance **Date:** Sanday, January 19, 2025
Banking and Insurance **Duration:** 1 hour and 30 minutes

Solution Exam of Actuarial

Theorical Exercise (05 points):

Why might an insurance company purchase reinsurance?............0.75
To reduce exposure to large losses, increase underwriting capacity, stabilize financial results, and protect against catastrophic events.

What is the importance of the Expected Value in insurance pricing?............0.75
Expected Value helps insurers estimate average claims costs, guiding premium pricing to cover potential losses while ensuring profitability.

Describe the role of probability distributions in risk modeling for insurance. How do actuaries choose an appropriate distribution for a given risk?.............01
Probability distributions model the likelihood of various outcomes, helping actuaries predict claim frequency and severity. The appropriate distribution is chosen based on the risk's characteristics, such as claim size, frequency, and available data.

Compare and contrast the use of Z-score and Z-credibility factor in the context of risk management. How do these tools complement each other in actuarial practices?...........01.25

- Z-score quantifies the deviation of individual outcomes, helping assess the risk of extreme events.

- Z-credibility factor adjusts risk estimates based on the reliability of the data.
Together, they provide a comprehensive view of risk by both identifying unusual outcomes and ensuring that predictions are based on credible data.

What is credibility theory, and how is it applied in insurance pricing? 01.25
Credibility theory is a statistical framework used in actuarial science to combine different sources of information (e.g., historical data and industry averages) to estimate the expected outcome for an individual risk.

It helps determine how much weight to assign to individual experience versus broader, more generalized data.

In insurance pricing, credibility theory is applied by adjusting premiums for individual policyholders or groups based on their own claims experience (specific data) and the collective experience (industry data).

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Exercise 1 (8 points):

1. **Calculate the Net Premium** for this 4-year term life insurance policy.

First, we calculate the Present Value of Death Benefit (PV Benefits)

The Net Present Value of the Death Benefit is given by:

$$\text{PV of Death Benefit} = \sum_{i=0}^{3} B.q_{45+t} v_{t+1} \text{................0.5}$$

Where:

B= 100,000 is the Death Benefit.

q_{45+t} is the probability of death at age 45+t.

$v_{t+1} = \frac{1}{(1+i)^{t+1}}$ is the discount factor at time t+1.

i = 0.06 is the annual interest rate.

Calculation for Year 0 (Age 45):

For age 45:

q_{45}=0.0012

$v_1 = \frac{1}{(1+0.06)^1} = 0.9433$.........0.5

Thus, the present value of the death benefit at age 45 is:

PV at age 45= 100,000 × 0.0012 × 0.9433 = 113.196..............0.5

- Continue the Calculation for the Next Years:

Now we repeat the calculation for each of the subsequent years (ages 46 to 48). The mortality probability and the discount factor change for each year, so be sure to use the correct values.

For age 46:

q_{46}=0.0013

$v_2 = \frac{1}{(1+0.06)^2} = 0.8899$.............0.5

So, the present value at age 46 would be:

PV at age 46= 100,000 ×0.0013 ×0.8899 = 115.687…………0.5

For age 47:

q_{47}=0.0014

$v_3 = \frac{1}{(1+0.06)^3} = 0.8396$…………..0.5

So, the present value at age 47 would be:

PV at age 47= 100,000 ×0.0014 ×0.8396 = 117.544…………0.5

For age 48:

q_{48}=0.0015

$v_4 = \frac{1}{(1+0.06)^4} = 0.7920$………………0.5

So, the present value at age 46 would be:

PV at age 48= 100,000 ×0.0016 ×0.7920= 126.72………………..0.5

Now you sum the present values for all 4 years:

PV of Death Benefit= PV at age 45+ PV at age 46+ PV at age 47+ PV at age 48

=113.196+115.687+117.544+126.72=473.147....................01

The Net Premium is the constant annual premium P paid for the first 4 years. The Present Value of Premiums is calculated by summing the present values of the premiums over the 4 years, with the discount factor applied to each year.

$$\text{Net Premium}=\frac{\text{PV of Death Benefit}}{\text{PV of Premiums}}$$

The Present Value of Premiums is given by:

$$\text{PV of Premiums} = P \times \sum_{i=1}^{4} v_t$$ ……………0.5

By calculation for each of the 4 years, summing the discounted values.

$\sum_{i=1}^{4} v_t = 0.9433 + 0.8899 + 0.8396 + 0.7920 = 3.4648$…………0.5

Thus, the present value of premium is

$$\text{PV of Premiums} = P \times 3.4648$$

- Solve for the Net Premium

To find the Net Premium P, we set the PV of Death Benefit equal to the PV of Premiums:

$$\text{PV of Death Benefit} = \text{PV of Premiums}$$

$$473.147 = P \times 3.4648$$

$$P = \frac{473.147}{3.4648} = 136.55$$ ………………..0.5

Thus, the premium is $136.55\$$

2- Calculate the Gross Premium

The Gross Premium is calculated as:

Gross Premium= Net Premium ×(1 + Expense Loading + Profit Margin)

$$= 136.55 \times (1 + 0.08 + 0.05) = 136.79 \times 1.13 = 154.3015$$ …………01

So, the Gross premium is 154.3015$.

<u>Exercise 2 (7 points)</u>:

1.Calculate the Expected Total Claims (S):

The total claims S is the sum of all individual claim amounts. Since the number of claims follows a **Poisson distribution**, and the claim amounts follow an **Exponential distribution**, we use the **Compound Distribution** model to find the expected total claims.

The **expected total claims** E(S) is calculated as:

$$E(S) = E(N) \cdot E(X)$$ …………01

Where:

- E(N) is the expected number of claims, which is the mean of the Poisson distribution ($\lambda=10$).
- E(X) is the expected claim amount, which is the mean of the Exponential distribution ($\beta=5{,}000$).

So,

$$E(S) = 10 \cdot 5{,}000 = 50{,}000 \text{ DZD}$$ ……………01

The expected total claims for the company in a year is **50,000 DZD**.

2.Calculate the Variance of the Total Claims (S):

The variance of the total claims S is calculated using the formula for the variance of a compound distribution:

Var(S)=E(N)·Var(X)+ E^2(X) ·Var(N)………………01

Where:

- Var(N) is the variance of the Poisson distribution, which is equal to the mean (λ=10).
- Var(X) is the variance of the Exponential distribution, which is equal to the square of the mean (β^2=5,000^2 = 25,000,000).

Substitute the values:

Var(S)=10·25,000,000+10·(5,000)^2

Var(S)=250,000,000+250,000,000

Var(S)=500,000,000 **DZD²**…………….01

The variance of the total claims is **500,000,000 DZD²**.

3. Determine the Probability of Total Claims Exceeding 100,000 DZD:

To calculate the probability of the total claims exceeding a certain amount (100,000 DZD in this case), we use the **Normal approximation** to the compound distribution………….0.5

For large values of λ, the distribution of the total claims S can be approximated by a normal distribution with the following parameters:

- Mean μ=E(S)=50,000 DZD.
- Variance σ^2 =Var(S)=500,000,000 DZD².
- Standard deviation σ=$\sqrt{500{,}000{,}000}$≈22,360.68 DZD……………..0.5

We want to find the probability P(S>100,000). Using the **z-score** formula:

z= $\frac{X-\mu}{\sigma}$………0.25

Where x = 100,000 DZD, μ=50,000 DZD, and σ≈22,360.68 DZD.

$$z = \frac{100{,}000 - 50{,}000}{22{,}360.68} \approx 2.24$$ ……………….0.75

Using the standard normal distribution table, the cumulative probability for z=2.24 is approximately **0.9875**. Therefore, the probability of total claims being less than 100,000 DZD is 0.9875.

To find the probability of exceeding 100,000 DZD:

$P(S>100{,}000)=1-P(S\leq 100{,}000)=1-0.9875=0.0125$………………….01

Thus, the probability of total claims exceeding 100,000 DZD in a year is approximately **1.25%**.