

Appendix 2

Tables of statistical distributions

Table A2.1 Normal critical values for different values of α

α	0.4	0.25	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.001
Z_{α}	.2533	.6745	.8416	1.0364	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902

Source: Author's computation using the NORMDIST function in Excel.

Table A2.2 Critical values of Student's t -distribution for different probability levels, α and degrees of freedom, ν

α	0.4	0.25	0.15	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
ν										
1	0.3249	1.0000	1.9626	3.0777	6.3138	12.7062	31.8205	63.6567	318.3087	636.6189
2	0.2887	0.8165	1.3862	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991
3	0.2767	0.7649	1.2498	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240
4	0.2707	0.7407	1.1896	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103
5	0.2672	0.7267	1.1558	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934	6.8688
6	0.2648	0.7176	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588
7	0.2632	0.7111	1.1192	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853	5.4079
8	0.2619	0.7064	1.1081	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008	5.0413
9	0.2610	0.7027	1.0997	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809
10	0.2602	0.6998	1.0931	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869
11	0.2596	0.6974	1.0877	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247	4.4370
12	0.2590	0.6955	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296	4.3178
13	0.2586	0.6938	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2208
14	0.2582	0.6924	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405
15	0.2579	0.6912	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728
16	0.2576	0.6901	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.0150
17	0.2573	0.6892	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651
18	0.2571	0.6884	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216
19	0.2569	0.6876	1.0655	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794	3.8834
20	0.2567	0.6870	1.0640	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518	3.8495
21	0.2566	0.6864	1.0627	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193
22	0.2564	0.6858	1.0614	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7921
23	0.2563	0.6853	1.0603	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7676
24	0.2562	0.6848	1.0593	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454
25	0.2561	0.6844	1.0584	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251
26	0.2560	0.6840	1.0575	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066
27	0.2559	0.6837	1.0567	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896
28	0.2558	0.6834	1.0560	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739
29	0.2557	0.6830	1.0553	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594
30	0.2556	0.6828	1.0547	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460
35	0.2553	0.6816	1.0520	1.3062	1.6896	2.0301	2.4377	2.7238	3.3400	3.5911
40	0.2550	0.6807	1.0500	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510
45	0.2549	0.6800	1.0485	1.3006	1.6794	2.0141	2.4121	2.6896	3.2815	3.5203
50	0.2547	0.6794	1.0473	1.2987	1.6759	2.0086	2.4033	2.6778	3.2614	3.4960
60	0.2545	0.6786	1.0455	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602
70	0.2543	0.6780	1.0442	1.2938	1.6669	1.9944	2.3808	2.6479	3.2108	3.4350
80	0.2542	0.6776	1.0432	1.2922	1.6641	1.9901	2.3739	2.6387	3.1953	3.4163
90	0.2541	0.6772	1.0424	1.2910	1.6620	1.9867	2.3685	2.6316	3.1833	3.4019
100	0.2540	0.6770	1.0418	1.2901	1.6602	1.9840	2.3642	2.6259	3.1737	3.3905
120	0.2539	0.6765	1.0409	1.2886	1.6577	1.9799	2.3578	2.6174	3.1595	3.3735
150	0.2538	0.6761	1.0400	1.2872	1.6551	1.9759	2.3515	2.6090	3.1455	3.3566
200	0.2537	0.6757	1.0391	1.2858	1.6525	1.9719	2.3451	2.6006	3.1315	3.3398
300	0.2536	0.6753	1.0382	1.2844	1.6499	1.9679	2.3388	2.5923	3.1176	3.3233
∞	0.2533	0.6745	1.0364	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905

Source: Author's own computation using the TINV function in Excel.

Table A2.3 Upper 5% critical values for F -distribution

		Degrees of freedom for numerator (m)																		
		1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
Degrees of freedom for denominator ($T - k$)																				
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254	
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

Source: Author's own computation using the Excel FINV function.

Table A2.4 Upper 1% critical values for F -distribution

Degrees of freedom for numerator (m)																			
1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞	
Degrees of freedom for denominator ($T - k$)																			
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,235	6,261	6,287	6,313	6,339	6,366
2	98.5	99.0	99.2	99.3	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.4	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Source: Author's own computation using the Excel FINV function.

Table A2.5 Chi-squared critical values for different values of α and degrees of freedom, ν

ν	0.995	0.990	0.975	0.950	0.900	0.750	0.500	0.250	0.100	0.050	0.025	0.010	0.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.1015	0.4549	1.323	2.706	3.841	5.024	6.635	7.879
2	0.01003	0.02010	0.05065	0.1026	0.2107	0.5754	1.386	2.773	4.605	5.991	7.378	9.210	10.597
3	0.07172	0.1148	0.2158	0.3518	0.5844	1.213	2.366	4.108	6.251	7.815	9.348	11.345	12.838
4	0.2070	0.2971	0.4844	0.7107	1.064	1.923	3.357	5.385	7.779	9.488	11.143	13.277	14.860
5	0.4117	0.5543	0.8312	1.145	1.610	2.675	4.351	6.626	9.236	11.070	12.833	15.086	16.750
6	0.6757	0.8721	1.237	1.635	2.204	3.455	5.348	7.841	10.645	12.592	14.449	16.812	18.548
7	0.9893	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	5.071	7.344	10.219	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.389	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342	12.549	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	7.584	10.341	13.701	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	8.438	11.340	14.845	18.54	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	9.299	12.340	15.984	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	10.165	13.339	17.117	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	11.036	14.339	18.245	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	11.912	15.338	19.369	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	12.792	16.338	20.489	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	13.675	17.338	21.605	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	14.562	18.338	22.718	27.204	30.143	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	15.452	19.337	23.828	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	16.344	20.337	24.935	29.615	32.670	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	17.240	21.337	26.039	30.813	33.924	36.781	40.289	42.796

Table A2.5 (cont.)

23	9.260	10.196	11.688	13.090	14.848	18.137	22.337	27.141	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	19.037	23.337	28.241	33.196	36.415	39.364	42.080	45.558
25	10.520	11.524	13.120	14.611	16.473	19.939	24.337	29.339	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	20.843	25.336	30.434	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	21.749	26.336	31.528	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	22.657	27.336	32.620	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	23.567	28.336	33.711	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	24.478	29.336	34.800	40.256	43.773	46.979	50.892	53.672
35	17.192	18.509	20.569	22.465	24.797	29.054	34.336	40.223	46.059	49.802	53.203	57.342	60.275
40	20.707	22.164	24.433	26.509	29.050	33.660	39.335	45.616	51.805	55.758	59.342	63.691	66.766
45	24.311	25.901	28.366	30.612	33.350	38.291	44.335	50.985	57.505	61.656	65.410	69.957	73.166
50	27.991	29.707	32.357	34.764	37.689	42.942	49.335	56.334	63.167	67.505	71.420	76.154	79.490
55	31.735	33.571	36.398	38.958	42.060	47.611	54.335	61.665	68.796	73.311	77.381	82.292	85.749
60	35.535	37.485	40.482	43.158	46.459	52.294	59.335	66.981	74.397	79.082	83.298	85.379	91.952
70	43.275	45.442	48.758	51.739	55.329	61.698	69.334	77.577	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	71.144	79.334	88.130	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	80.625	89.334	98.650	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	90.133	99.334	109.141	118.498	124.342	129.561	135.807	140.169
120	83.829	86.909	91.568	95.705	100.627	109.224	119.335	130.051	140.228	146.565	152.214	158.963	163.670
150	109.122	112.655	117.980	122.692	126.278	137.987	149.334	161.258	172.577	179.579	185.803	193.219	198.380
200	152.224	156.421	162.724	168.279	174.825	156.175	199.334	213.099	226.018	233.993	241.060	249.455	255.281
250	196.145	200.929	208.095	214.392	221.809	234.580	249.334	264.694	279.947	287.889	295.691	304.948	311.361

Source: Author's own computation using the Excel CHINV function.

Table A2.6 Lower and upper 1% critical values for Durbin–Watson statistic

T	$k' = 1$		$k' = 2$		$k' = 3$		$k' = 4$		$k' = 5$	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	0.81	1.07	0.70	1.25	0.59	1.46	0.49	1.70	0.39	1.96
16	0.84	1.09	0.74	1.25	0.63	1.44	0.53	1.66	0.44	1.90
17	0.87	1.10	0.77	1.25	0.67	1.43	0.57	1.63	0.48	1.85
18	0.90	1.12	0.80	1.26	0.71	1.42	0.61	1.60	0.52	1.80
19	0.93	1.13	0.83	1.26	0.74	1.41	0.65	1.58	0.56	1.77
20	0.95	1.15	0.86	1.27	0.77	1.41	0.68	1.57	0.60	1.74
21	0.97	1.16	0.89	1.27	0.80	1.41	0.72	1.55	0.63	1.71
22	1.00	1.17	0.91	1.28	0.83	1.40	0.75	1.54	0.66	1.69
23	1.02	1.19	0.94	1.29	0.86	1.40	0.77	1.53	0.70	1.67
24	1.04	1.20	0.96	1.30	0.88	1.41	0.80	1.53	0.72	1.66
25	1.05	1.21	0.98	1.30	0.90	1.41	0.83	1.52	0.75	1.65
26	1.07	1.22	1.00	1.31	0.93	1.41	0.85	1.52	0.78	1.64
27	1.09	1.23	1.02	1.32	0.95	1.41	0.88	1.51	0.81	1.63
28	1.10	1.24	1.04	1.32	0.97	1.41	0.90	1.51	0.83	1.62
29	1.12	1.25	1.05	1.33	0.99	1.42	0.92	1.51	0.85	1.61
30	1.13	1.26	1.07	1.34	1.01	1.42	0.94	1.51	0.88	1.61
31	1.15	1.27	1.08	1.34	1.02	1.42	0.96	1.51	0.90	1.60
32	1.16	1.28	1.10	1.35	1.04	1.43	0.98	1.51	0.92	1.60
33	1.17	1.29	1.11	1.36	1.05	1.43	1.00	1.51	0.94	1.59
34	1.18	1.30	1.13	1.36	1.07	1.43	1.01	1.51	0.95	1.59
35	1.19	1.31	1.14	1.37	1.08	1.44	1.03	1.51	0.97	1.59
36	1.21	1.32	1.15	1.38	1.10	1.44	1.04	1.51	0.99	1.59
37	1.22	1.32	1.16	1.38	1.11	1.45	1.06	1.51	1.00	1.59
38	1.23	1.33	1.18	1.39	1.12	1.45	1.07	1.52	1.02	1.58
39	1.24	1.34	1.19	1.39	1.14	1.45	1.09	1.52	1.03	1.58
40	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
45	1.29	1.38	1.24	1.42	1.20	1.48	1.16	1.53	1.11	1.58
50	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
55	1.36	1.43	1.32	1.47	1.28	1.51	1.25	1.55	1.21	1.59
60	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
65	1.41	1.47	1.38	1.50	1.35	1.53	1.31	1.57	1.28	1.61
70	1.43	1.49	1.40	1.52	1.37	1.55	1.34	1.58	1.31	1.61
75	1.45	1.50	1.42	1.53	1.39	1.56	1.37	1.59	1.34	1.62
80	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
85	1.48	1.53	1.46	1.55	1.43	1.58	1.41	1.60	1.39	1.63
90	1.50	1.54	1.47	1.56	1.45	1.59	1.43	1.61	1.41	1.64
95	1.51	1.55	1.49	1.57	1.47	1.60	1.45	1.62	1.42	1.64
100	1.52	1.56	1.50	1.58	1.48	1.60	1.46	1.63	1.44	1.65

Note: T , number of observations; k' , number of explanatory variables (excluding a constant term).
Source: Durbin and Watson (1951): 159–77. Reprinted with the permission of Oxford University Press.

Table A2.7 Dickey–Fuller critical values for different significance levels, α

Sample size T	0.01	0.025	0.05	0.10
τ				
25	−2.66	−2.26	−1.95	−1.60
50	−2.62	−2.25	−1.95	−1.61
100	−2.60	−2.24	−1.95	−1.61
250	−2.58	−2.23	−1.95	−1.62
500	−2.58	−2.23	−1.95	−1.62
∞	−2.58	−2.23	−1.95	−1.62
τ_μ				
25	−3.75	−3.33	−3.00	−2.63
50	−3.58	−3.22	−2.93	−2.60
100	−3.51	−3.17	−2.89	−2.58
250	−3.46	−3.14	−2.88	−2.57
500	−3.44	−3.13	−2.87	−2.57
∞	−3.43	−3.12	−2.86	−2.57
τ_τ				
25	−4.38	−3.95	−3.60	−3.24
50	−4.15	−3.80	−3.50	−3.18
100	−4.04	−3.73	−3.45	−3.15
250	−3.99	−3.69	−3.43	−3.13
500	−3.98	−3.68	−3.42	−3.13
∞	−3.96	−3.66	−3.41	−3.12

Source: Fuller (1976). Reprinted with the permission of John Wiley and Sons.

Table A2.8 Critical values for the Engle–Granger cointegration test on regression residuals with no constant in test regression

Number of variables in system	Sample size T	0.01	0.05	0.10
2	50	−4.32	−3.67	−3.28
	100	−4.07	−3.37	−3.03
	200	−4.00	−3.37	−3.02
3	50	−4.84	−4.11	−3.73
	100	−4.45	−3.93	−3.59
	200	−4.35	−3.78	−3.47
4	50	−4.94	−4.35	−4.02
	100	−4.75	−4.22	−3.89
	200	−4.70	−4.18	−3.89
5	50	−5.41	−4.76	−4.42
	100	−5.18	−4.58	−4.26
	200	−5.02	−4.48	−4.18

Source: Engle and Yoo (1987). Reprinted with the permission of Elsevier.

Table A2.9 Quantiles of the asymptotic distribution of the Johansen cointegration rank test statistics (constant in cointegrating vectors only)

$p - r$	50%	80%	90%	95%	97.5%	99%	Mean	Var
$\lambda_{\max}$								
1	3.40	5.91	7.52	9.24	10.80	12.97	4.03	7.07
2	8.27	11.54	13.75	15.67	17.63	20.20	8.86	13.08
3	13.47	17.40	19.77	22.00	24.07	26.81	14.02	19.24
4	18.70	22.95	25.56	28.14	30.32	33.24	19.23	23.83
5	23.78	28.76	31.66	34.40	36.90	39.79	24.48	29.26
6	29.08	34.25	37.45	40.30	43.22	46.82	29.72	34.63
7	34.73	40.13	43.25	46.45	48.99	51.91	35.18	38.35
8	39.70	45.53	48.91	52.00	54.71	57.95	40.35	41.98
9	44.97	50.73	54.35	57.42	60.50	63.71	45.55	44.13
10	50.21	56.52	60.25	63.57	66.24	69.94	50.82	49.28
11	55.70	62.38	66.02	69.74	72.64	76.63	56.33	54.99
λ_{Trace}								
1	3.40	5.91	7.52	9.24	10.80	12.97	4.03	7.07
2	11.25	15.25	17.85	19.96	22.05	24.60	11.91	18.94
3	23.28	28.75	32.00	34.91	37.61	41.07	23.84	37.98
4	38.84	45.65	49.65	53.12	56.06	60.16	39.50	59.42
5	58.46	66.91	71.86	76.07	80.06	84.45	59.16	91.65
6	81.90	91.57	97.18	102.14	106.74	111.01	82.49	126.94
7	109.17	120.35	126.58	131.70	136.49	143.09	109.75	167.91
8	139.83	152.56	159.48	165.58	171.28	177.20	140.57	208.09
9	174.88	198.08	196.37	202.92	208.81	215.74	175.44	257.84
10	212.93	228.08	236.54	244.15	251.30	257.68	213.53	317.24
11	254.84	272.82	282.45	291.40	298.31	307.64	256.15	413.35

Source: Osterwald-Lenum (1992, table 1*). Reprinted with the permission of Blackwell Publishers.

Table A2.10 Quantiles of the asymptotic distribution of the Johansen cointegration rank test statistics (constant, i.e. a drift only in VAR and in cointegrating vector)

$p - r$	50%	80%	90%	95%	97.5%	99%	Mean	Var
λ_{max}								
1	0.44	1.66	2.69	3.76	4.95	6.65	0.99	2.04
2	6.85	10.04	12.07	14.07	16.05	18.63	7.47	12.42
3	12.34	16.20	18.60	20.97	23.09	25.52	12.88	18.67
4	17.66	21.98	24.73	27.07	28.98	32.24	18.26	23.47
5	23.05	27.85	30.90	33.46	35.71	38.77	23.67	28.82
6	28.45	33.67	36.76	39.37	41.86	45.10	29.06	33.57
7	33.83	39.12	42.32	45.28	47.96	51.57	34.37	37.41
8	39.29	45.05	48.33	51.42	54.29	57.69	39.85	42.90
9	44.58	50.55	53.98	57.12	59.33	62.80	45.10	44.93
10	49.66	55.97	59.62	62.81	65.44	69.09	50.29	49.41
11	54.99	61.55	65.38	68.83	72.11	75.95	55.63	54.92
λ_{Trace}								
1	0.44	1.66	2.69	3.76	4.95	6.65	0.99	2.04
2	7.55	11.07	13.33	15.41	17.52	20.04	8.23	14.38
3	18.70	23.64	26.79	29.68	32.56	35.65	19.32	32.43
4	33.60	40.15	43.95	47.21	50.35	54.46	34.24	52.75
5	52.30	60.29	64.84	68.52	71.80	76.07	52.95	79.25
6	75.26	84.57	89.48	94.15	98.33	103.18	75.74	114.65
7	101.22	112.30	118.50	124.24	128.45	133.57	101.91	158.78
8	131.62	143.97	150.53	156.00	161.32	168.36	132.09	201.82
9	165.11	178.90	186.39	192.89	198.82	204.95	165.90	246.45
10	202.58	217.81	225.85	233.13	239.46	247.18	203.39	300.80
11	243.90	260.82	269.96	277.71	284.87	293.44	244.66	379.56

Source: Osterwald-Lenum (1992, table 1). Reprinted with the permission of Blackwell Publishers.

Table A2.11 Quantiles of the asymptotic distribution of the Johansen cointegration rank test statistics (constant in cointegrating vector and VAR, trend in cointegrating vector)

$p - r$	50%	80%	90%	95%	97.5%	99%	Mean	Var
λ_{max}								
1	5.55	8.65	10.49	12.25	14.21	16.26	6.22	10.11
2	10.90	14.70	16.85	18.96	21.14	23.65	11.51	16.38
3	16.24	20.45	23.11	25.54	27.68	30.34	16.82	22.01
4	21.50	26.30	29.12	31.46	33.60	36.65	22.08	27.74
5	26.72	31.72	34.75	37.52	40.01	42.36	27.32	31.36
6	32.01	37.50	40.91	43.97	46.84	49.51	32.68	37.91
7	37.57	43.11	46.32	49.42	51.94	54.71	38.06	39.74
8	42.72	48.56	52.16	55.50	58.08	62.46	43.34	44.83
9	48.17	54.34	57.87	61.29	64.12	67.88	48.74	49.20
10	53.21	59.49	63.18	66.23	69.56	73.73	53.74	52.64
11	58.54	64.97	69.26	72.72	75.72	79.23	59.15	56.97
λ_{Trace}								
1	5.55	8.65	10.49	12.25	14.21	16.26	6.22	10.11
2	15.59	20.19	22.76	25.32	27.75	30.45	16.20	24.90
3	29.53	35.56	39.06	42.44	45.42	48.45	30.15	45.68
4	47.17	54.80	59.14	62.99	66.25	70.05	47.79	74.48
5	68.64	77.83	83.20	87.31	91.06	96.58	69.35	106.56
6	94.05	104.73	110.42	114.90	119.29	124.75	94.67	143.33
7	122.87	134.57	141.01	146.76	152.52	158.49	123.51	182.85
8	155.40	169.10	176.67	182.82	187.91	196.08	156.41	234.11
9	192.37	207.25	215.17	222.21	228.05	234.41	193.03	288.30
10	231.59	247.91	256.72	263.42	270.33	279.07	232.25	345.23
11	276.34	294.12	303.13	310.81	318.02	327.45	276.88	416.98

Source: Osterwald-Lenum (1992, table 2*). Reprinted with the permission of Blackwell Publishers.